Optimal Lunar ISRU Plant Deployment Under Oxygen Demand Uncertainty Using Model Predictive Control

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1. Introduction

The optimal deployment of lunar ISRU plants is crucial to realize an efficient lunar ISRU. Identifying the best strategy for deployment is, however, challenging due to significant uncertainties in the lunar environment and ISRU operations.

Uncertainty examples related to lunar ISRU:

- **Liquid OXygen (LOX) demand**,
- Lunar regolith content,
- Sunlight availability.

Decision-making without considering LOX demand uncertainty can lead to a risk of insufficient or unnecessarily excessive production capacity. Unfortunately, precise LOX demand prediction remains impossible due to a lack of past data. To address this uncertainty, this research proposes using model predictive control (MPC) to optimize lunar ISRU deployment strategies.

2. Model Predictive Control

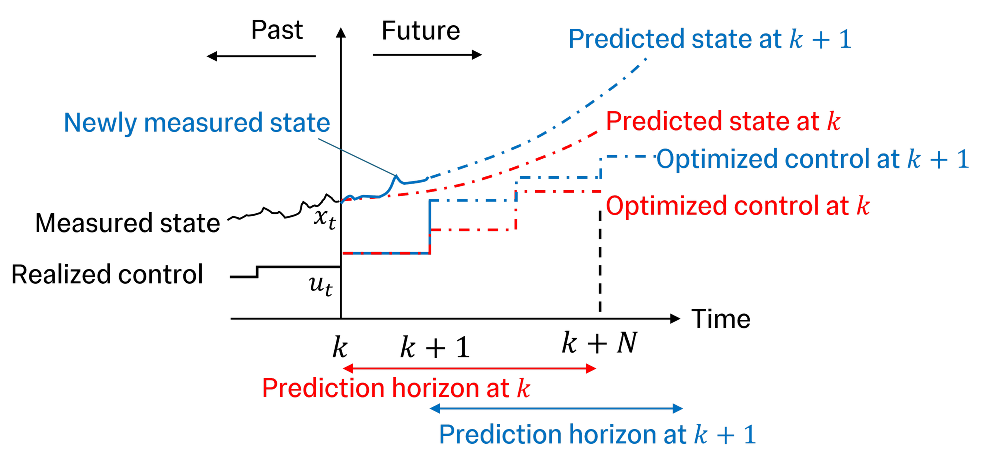

Figure 1: MPC scheme. at time k MPC optimizes a series of control for future N steps and implement the first one.

MPC is a control scheme used in process industries (e.g., chemical plants). MPC optimizes a series of controls (i.e., decisions) for a finite time horizon (N steps). The first of the optimal control is applied to the system, then MPC measures the state and re-optimizes the control again.

3. Preliminary Case Study

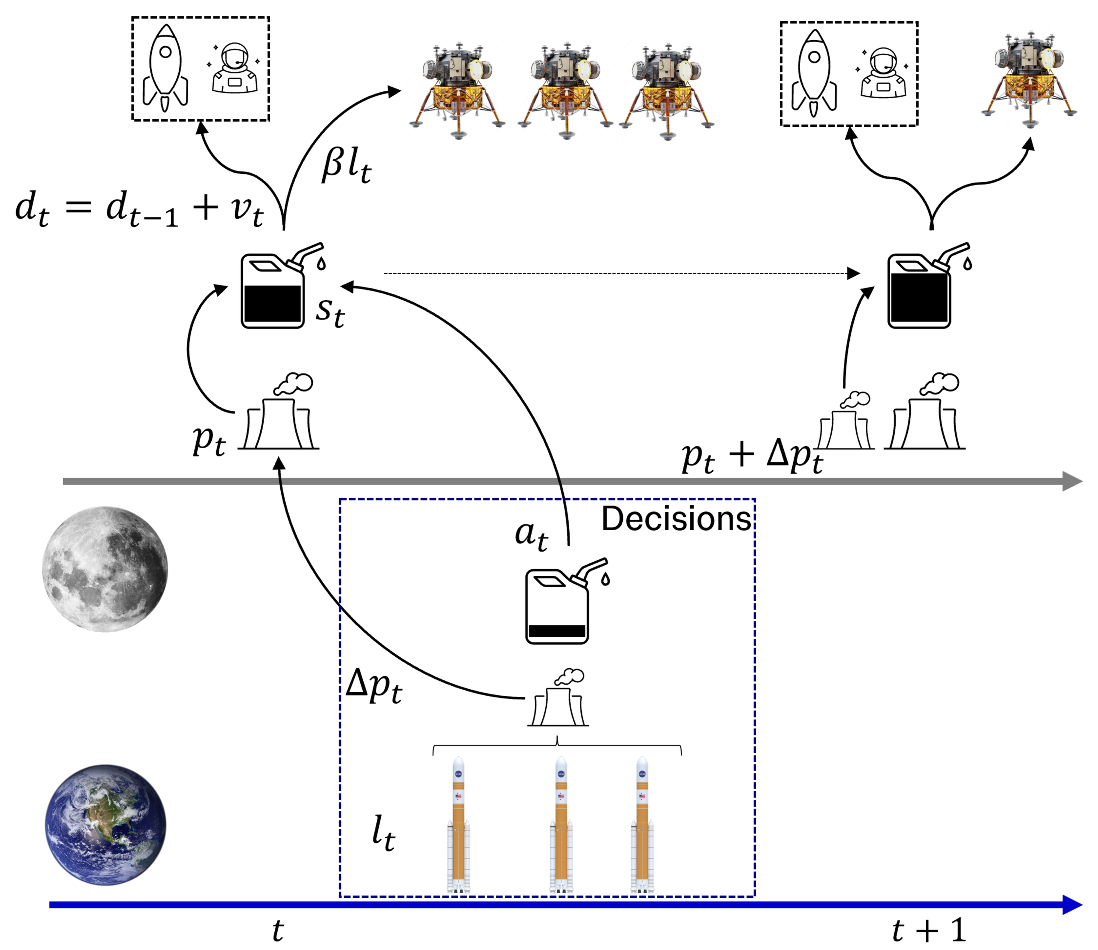

Figure 2: Case study. Each time a decision maker decide how much we expand the production capacity and how much LOX we directly bring to the Moon.

Problem Formulations

This case study demonstrates the use of single-objective robust MPC to minimize the landed mass under LOX demand uncertainty. A state x and the control vector u at time t is represented as:

$$x_t = \begin{bmatrix} p\\ s\\ d\end{bmatrix}_t = \begin{bmatrix}\text{LOX prod.capacity}\\ \text{LOX stock}\\ \text{LOX demand}\end{bmatrix}_t, \quad (1)$$

$$u_t = \begin{bmatrix} \Delta p\\ a\\ l\end{bmatrix}_t = \begin{bmatrix}\text{Additional LOX prod.capacity}\\ \text{Direct LOX import from Earth}\\ \text{Number of rocket launches}\end{bmatrix}_t. \quad (2)$$

Then, the state transition can be expressed as follows:

$$x_{t+1} = \begin{bmatrix}1&0&0\\1&\alpha&-1\\0&0&\varphi\end{bmatrix} x_t + \begin{bmatrix}1&0&0\\1&1&-\beta\\0&0&0\end{bmatrix} u_t + \begin{bmatrix}0\\0\\1\end{bmatrix} v_t, \quad (3)$$

where the demand is modeled as a first-order autoregressive model $AR(1)$ with a parameter $\varphi = 1$. The LOX boil-off rate and required LOX mass for a lander's one trip is expressed as α and β, respectively.

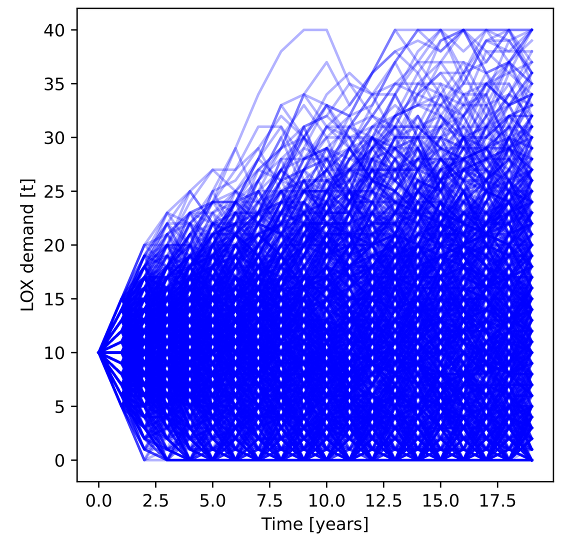

Figure 3: LOX demand scenarios tested in this case study. The time horizon is set to 20 years.

The optimization problem can be formulated as follows: minimize

$$\sum_{t=k}^{k+N-1}\left(\begin{bmatrix} m_{\text{reac}}&0&0\\0&0&0\\0&0&0\end{bmatrix} x_t + \begin{bmatrix} m_{\text{reac}}+m_{\text{ISRU}}&0&0\\1&m_{\text{tank}}&0\\0&0&0\end{bmatrix} u_t\right) \quad (4)$$

subject to

$$\text{Eq. 3},$$

$$\begin{bmatrix}\gamma & \zeta & -c\end{bmatrix} u_t \le 0, \quad (5)$$

$$p_t, s_t, d_t, \Delta p_t, a_t, l_t \ge 0 \quad \forall t \in \mathcal{T}. \quad (6)$$

Inequality 5 expresses rocket launch capacity.

Results

Using the robust MPC approach [1], the MPC problem above can be solved. We further compared the result with a simple decision rule approach. The decision rule used here states: "If the LOX stock s is smaller than 5t for two consecutive years, then expand the production capacity by 10 t." Figure 4 illustrates that the robust MPC approach reduces the expected landed mass by 5 t. Figure 5 shows that the expected failure (time periods when $s=0$) rate is reduced approximately by 10 %.

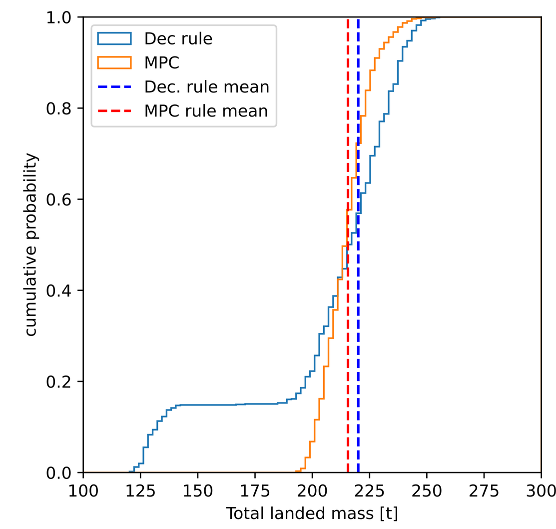

Figure 4: Cumulative probabilities of total landed mass for each method. MPC can deploy ISRU plants slightly better in terms of landed mass, while the decision rule approach can lead to smaller landed mass in some cases.

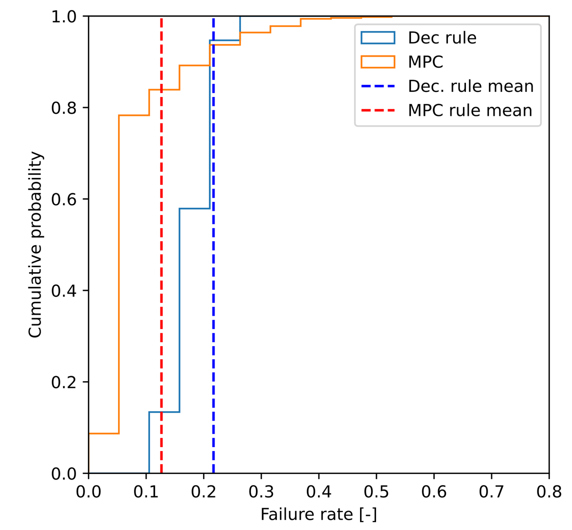

Figure 5: Cumulative probabilities of failure rate. The failure rate is defined as the ratio of time periods that LOX stock $s=0$ to the studied time horizon.

4. Future Work and Summary

As Ikeya et al., [2] suggested, considering multiple objectives is crucial in ISRU deployment. Future research should focus on expanding to Multi-objective Model Predictive Control (MOMPC). Additionally, exploring other MPC methods, such as distributionally robust MPC, could provide valuable insights.

Summary:

- **MPC** is used to optimize lunar ISRU deployment under uncertainty,
- The result shows its better performance compared to the decision rule approach,
- Expansion to MOMPC should be studied.

References

[1] Felix Fiedler et al. "Do-Mpc: Towards FAIR Nonlinear and Robust Model Predictive Control". Nov. 2023.

[2] Kosuke Ikeya et al. Multi-Objective Decision Analyses on Deploying Lunar In-Situ Resource Utilization Plants Under Resource and Operational Uncertainty. Oct. 2024.

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