**OPTIMAL LUNAR ISRU PLANT DEPLOYMENT UNDER OXYGEN DEMAND UNCERTAINTY USING MODEL PREDICTIVE CONTROL.**

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**Introduction:** The optimal deployment of lunar in-situ resource utilization (ISRU) plants is crucial to successfully realize an efficient and reasonable lunar ISRU and ultimately establish a self-sustained human presence on and around the Moon. Identifying the best deployment strategy is, however, challenging due to large uncertainty related to the lunar environment as well as the ISRU operation. Among many uncertain factors, the demand for Liquid OXygen (LOX) is one of the most critical yet unpredictable parameters. Past studies optimized the lunar ISRU deployment under uncertainty [1], but they have often overlooked LOX demand. Decision-making without considering this uncertainty can lead to a risk of insufficient or unnecessarily excessive production capacity. Unfortunately, predicting LOX demand in the future is, if not impossible, challenging due to a lack of past data. To deal with this extreme uncertainty, this research proposes using model predictive control (MPC) to solve a sequential decision problem. MPC generates the best decisions adapting to (potentially) fluctuating LOX demand. The optimal decisions are made under 1000 different demand realizations, and compared against a rather traditional decision rule approach.

**Methodology:** Figure 1 depicts the methodology employed in this paper based on Multi-Objective MPC (MOMPC). At each time instant, MPC optimizes a series of controls (or decisions) for a finite future time horizon ($N$ steps in this paper) based on the current state. Since this is a multi-objective approach, a Pareto optimal set can be generated. A decision maker can select a series of decisions to be implemented from the Pareto set following the criteria at that time. Once the first decision is implemented, the system observes an actual realization of an uncertain future. The optimization process is then repeated using the updated state.

As a preliminary study, this project uses a single-objective non-conservative robust MPC technique proposed by Lucia et al.[2] An extension into a multi-objective problem could be done using a weight matrix [3].

For a linear system, this method solves the following optimization at a time $t = k$:

$$\min_{\boldsymbol{U}_k} J(x_k, \boldsymbol{U}_k) \tag{1}$$

subject to:

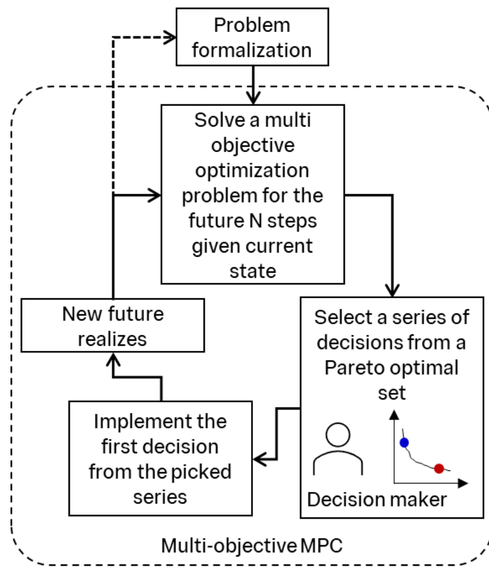


*Figure 1 –Proposed methodology using multi-objective MPC for sequential decision making.*

$$x_{k+1} = Ax_k + Bu_k + Dw_k \tag{2a}$$
$$\boldsymbol{U}_k = [u_k, u_{k+1}, \cdots, u_{k+N-1}] \tag{2b}$$
$$x_k \in \mathcal{X}\ \forall k \tag{2c}$$
$$u_k \in \mathcal{U}\ \forall k, \tag{2d}$$

where $x_k$, $u_k$, $w_k$ represent the state, control, and uncertain disturbance vectors at $t = k$, respectively. The state and input constraints are expressed in (2c) and (2d).

**Case Study:**

*Overview and Assumptions.* We use the proposed method for a simple case study. In this preliminary case study, the deployment of lunar ISRU carbothermal reduction (CR) plants under LOX uncertainty is considered.

It is assumed that each capacity expansion is done by deploying an additional system with all necessary components from regolith excavation to liquefaction, instead of adding additional components such as reactors.

*System.* The state, control and uncertain disturbance of this case study are represented as $x_t = [c_t \quad s_t \quad d_t]^T$, $u_t = [\Delta c_t \quad a_t]^T$, and $W_t = \delta d_t$, respectively. The elements in the state vector $c, s, d$ represent the LOX production capacity, LOX stock on the

Moon, and the demand for the LOX. The additional capacity to be installed on the Moon is denoted by $\Delta c$, while $a$ represents the mass of the additional LOX directly imported from Earth. Furthermore, the change in the demand is denoted by $\delta d$.

A simple lunar ISRU deployment problem can be represented as follows:

$$\begin{bmatrix} c \\ s \\ d \end{bmatrix}_{t+1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ s \\ d \end{bmatrix}_t + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta c \\ a \end{bmatrix}_t + \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \delta d_t. \tag{3}$$

As a preliminary case study, the objective is set to minimize the landed mass, which can be expressed as follows:

$$J = \sum_{t \in \mathcal{T}} \left( M^I \delta c_t + a_t + M^r (c_t + \delta c_t) \right). \tag{3}$$

Based on [4], the landed mass of a CR ISRU plant has an almost linear dependency on the capacity, and $M^I \delta c_t$ represents the mass of the system deployed at that time. The CR architecture also requires additional reactants to compensate for imperfect reactant recycling. This mass depends on the total capacity, and thus, is represented by $M^r (c_t + \delta c_t)$.

The LOX demand is modeled as a random walk. The change in the demand from the last year is assumed to be $\pm$ 5 t with a uniform distribution with an initial demand of 10 t. Figure 2 shows 1000 different scenarios of the LOX considered in this case study.

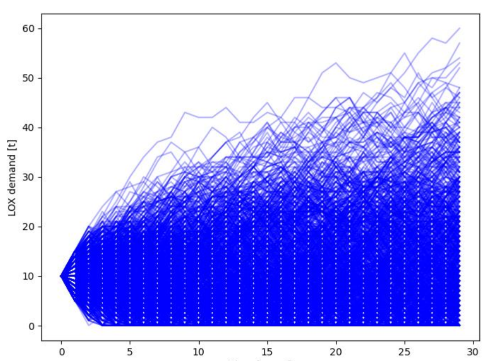


*Figure 2 –1000 realizations of LOX annual demand properties in 30 years.*

*Preliminary results.* Figures 3 and 4 compare the performance of the proposed MPC-based approach with a more conventional decision rule approach. The employed decision rule is "IF the LOX stock is lower than a threshold value of 2 t for two consecutive years, THEN expand the production capacity by 10 t. OTHERWISE do nothing." As can be seen in Fig. 4, the expected landed mass following the proposed method is smaller than following the decision rule. The proposed method, however, generated a larger landed mass at the 95th percentile, indicating less favorable for risk-averse decision-makers.

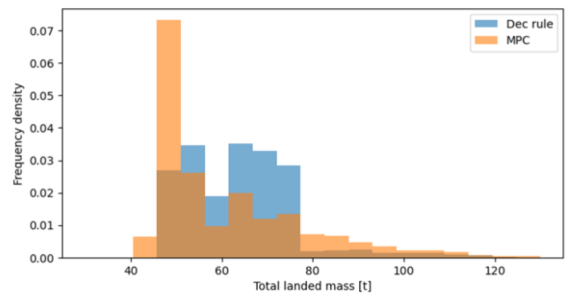


*Figure 3 – Histograms of total landed mass using the decision-rule approach and MPC.*

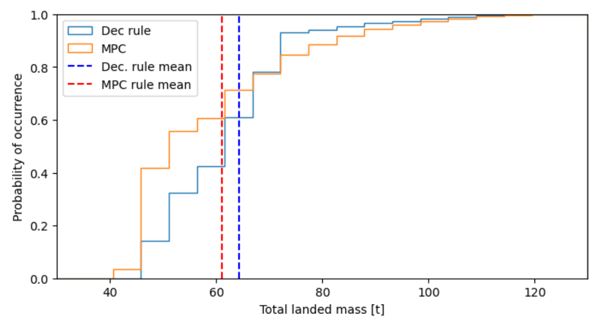


*Figure 4 – Cumurative probabilities of total landed mass using the decision-rule approach and MPC.*

**Conclusion:** This paper has proposed a new MPC-based method to assist decision-makers in making decisions under uncertainty. The simple case study shows its potential. To incorporate deep uncertainty consideration, a more sophisticated MPC method such as a distributionally robust MPC should be explored in the future.

**References:** [1] K. Ikeya et al. (2024) *IAC-24*,A3,IP,149,x85667. [2] S. Lucia et al. (2013) *J. Process Contr.*, *23*, 1306-1319. [3] A. Bemporad and D. de la Peña (2009) *Automatica*, *45*, 2823-2830. [4] K. Ikeya et al. (2025) *Acta Astronaut.*, *230*, 148–168.